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On disclosure policy in contests with stochastic entry

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Abstract We study how a contest organizer who seeks to maximize participant effort should disclose the information on the actual number of contestants in an imperfectly discriminatory contest with stochastic entry. When each potential contestant has a fixed probability of entering the contest, the optimal disclosure policy depends crucially on the properties of the characteristic function $H(\cdot) = f(\cdot)/f'(\cdot)$, where $f(\cdot)$ is the impact function. The contest organizer prefers full disclosure (full concealment) if $H(\cdot)$ is strictly concave (strictly convex). However, the expected equilibrium effort is independent of the prevailing information disclosure policy if a linear $H(\cdot)$ (Tullock Contest) applies.

Keywords Contests · Stochastic entry · Number of contestants · Disclosure · Effort

JEL Classification C72 · D72 · D82

1 Introduction

Much of the contest literature makes the assumption that the number of competing agents is fixed, and that this number is known by all participants. Although this paradigm simplifies the analysis significantly, it stands in contrast to numerous contest settings in real-life that

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involve an uncertain set of participants. For instance, a firm racing to develop an innovation may not know how many other firms are pursuing the same idea. Similarly, a job applicant may be uncertain about the number of competitors for the same post. In a procurement tournament, a seller may not be aware of the number of bidders who are interested in the contract.

In this study, we consider contests with a stochastic number of contestants. Our setting involves a fixed number of potential contestants, each of whom has a fixed probability of entering the contest. The realized number of participants remains uncertain, but follows a binomial distribution. The participating contestants exert costly and nonrefundable efforts to compete for a single prize. We further assume that their effort accrues to the benefit of the contest organizer. In this scenario, our analysis sets out to address a classical question in the contest literature: How does the contest organizer choose a disclosure policy that maximizes the expected total effort? That is, Should the contest organizer disclose or conceal the actual number of contestants to participants? Which policy alternative leads to a higher level of expected total effort?

To address these questions, we consider a three-stage game. In the first stage, the contest organizer chooses her disclosure policy. She either reveals the actual number of contestants, or conceals this information. She announces her policy choice publicly to potential contestants. In the second stage, the actual number of contestants is realized and learnt by the organizer. This information is disclosed to the contestants if the organizer had earlier chosen to do so. In the third stage, contestants submit their effort entries simultaneously in competition for the single prize.

We adopt the well-studied ratio-form contest success function to abstract the underlying stochastic winner selection process.[1] In this setting, a contestant i, who exerts an effort x_i, wins the prize with a probability $p_i(x_i, \mathbf{x}_{-i}) = \frac{f(x_i)}{f(x_i)+\sum_{j\neq i} f(x_j)}$ if there are $N-1$ others who exert effort of $\mathbf{x}_{-i} = (x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N)$. The function $f(\cdot)$ has been named the "impact function" by Wärneryd (2001), and it specifies each contestant's production technology in the contest.

The optimal disclosure policy depends crucially on the *characteristic function* of the contest, which is formally defined as $H(x) \equiv \frac{f(x)}{f'(x)}$. The properties of this function determine how each participating contestant responds to various environmental factors in the contest. We show that disclosing the actual number of contestants leads to a higher (lower) level of total effort, relative to concealing the information, if the characteristic function is concave (convex). However, the level of expected total effort is independent of the prevailing disclosure policy, if the characteristic function is linear. We further show that a linear characteristic function is uniquely generated by contests known as Tullock (1980) contests, which assume $f(x) = x^r$.

Our analysis yields interesting theoretical implications. Despite all contestants being risk-neutral, a strictly concave characteristic function leads contestants to behave as if they were risk-loving when they supply their effort.[2] Conversely, "pseudo" risk-aversion appears when a strictly convex characteristic function applies. With non-Tullock contest technologies, the disclosure policy plays a pivotal role in determining the equilibrium level of effort, because of the "pseudo" risk-loving/averse attitudes that are underpinned by concave/convex characteristic functions.

[1] The reader is referred to Skaperdas (1996) for the axiomatic foundation of the ratio-form contest success function and Fu and Lu (2008) for the function's micro-foundation that is derived from a noisy-ranking perspective.

[2] In other words, the individual effort function is convex in terms of the amount of prize.

To check the robustness of our main results and to deepen our analysis, we further generalize our basic setting by allowing the contest organizer to partially disclose the actual number of participants. Under a partial disclosure policy, the organizer does not reveal the exact number of participants, but only the range of this number. *Will the organizer benefit from partial disclosure? How should she structure the optimal partial disclosure policy?* We show that strict concavity (convexity) of the characteristic function must lead to full disclosure (full concealment), and partial disclosure is never optimal. By way of contrast, the disclosure policy does not affect the expected overall effort in a Tullock contest (which has a linear characteristic function), in spite of the numerous possible ways of constructing a partial disclosure policy.

Only a handful of papers have formally investigated contests with stochastic participation. Higgins et al. (1985) pioneered this strand of literature by studying a contest in which each rent seeker bears a fixed cost for participation. They established a unique symmetric mixed strategy equilibrium, where each rent seeker randomly enters the contest, and ends up with zero surplus. While Higgins et al. (1985) investigated endogenous entry strategies, a few other studies have assumed exogenous entry patterns. Myerson and Wärneryd (2006) examined a contest with an infinite number of potential entrants. Both Münster (2006) and Lim and Matros (2010) assumed a finite pool of potential contestants. In their setting, each participating contestant enters the contest with a fixed and independent probability and the number of participating contestants follows a binomial distribution. Münster (2006) focused on the impact of players' risk attitudes on the contestants' incentive to supply effort. In contrast, Lim and Matros (2010) considered a scenario with risk-neutral contestants.

The current study is most closely related to Lim and Matros (2010), who provide a complete account of the bidding equilibrium in a Tullock contest with a stochastic number of contestants. To the best of our knowledge, Lim and Matros (2010) are the first to study optimal disclosure policies in contests. They establish that the disclosure policy (full disclosure or full concealment) does not impact the level of effort. Our analysis allows for more general contest technologies, and we find sharply different results that indicate the "relevance" of disclosure policy when non-Tullock contests are considered. Furthermore, we allow contest organizers to partially disclose information. The "disclosure irrelevance" principle in Tullock contests (with their linear characteristic functions) holds, despite the substantially richer set of candidate disclosure strategies available to organizers. Our study thus complements Lim and Matros (2010) in these regards.

2 Contest with a stochastic number of contestants

Let M (≥ 2) denote the set of risk neutral potential contestants whose probability of participating in the contest is $q \in (0, 1)$. All participating contestants compete for a single prize of value $v > 0$.

Suppose that $N \leq M$ contestants participate and simultaneously commit to their nonnegative rent seeking efforts $x_i, i = 1, 2, \dots, N$. The effort is costly and non-refundable, and the contestants incur a unit marginal cost. We assume also that the winner is determined by a ratio-form contest success function. This mechanism has been commonly adopted in the literature, and is axiomatized by Skaperdas (1996). If $N \geq 2$ contestants enter the contest, a participating contestant i wins the prize v with a probability

$$p_i(x_i, \mathbf{x}_{-i}; N) = \frac{f(x_i)}{\sum_{j=1}^{N} f(x_j)}, \tag{1}$$

where the function $f(\cdot)$ is strictly increasing, thrice differentiable and weakly log concave, with $f(0)=0$. The log-concavity, as will be shown, guarantees the uniqueness of symmetric equilibrium in the contest. Wärneryd (2001) names $f(\cdot)$ the impact function of the contest, which indicates a contestant's production technology. If all contestants make zero effort, we assume that the prize recipient is randomly chosen from the pool. Moreover, we assume that if there is only one participant, then he automatically wins the prize regardless of his effort.

We assume further that the effort exerted by the contestants accrues to the benefit of the contest organizer. The contest organizer is allowed to commit to her disclosure policy—either to disclose the actual number of participants, or to conceal this information—and announces this policy choice publicly. We denote the former policy by D, and the latter by C. Nature then determines N, the actual number of participants. The organizer observes this information, and discloses it if and only if she has committed to a disclosure. The participants effort entries simultaneously $\mathbf{x}=(x_i)$ to compete for the prize.

2.1 Equilibrium

We now explore the equilibrium of the contest under each policy. We first consider a case in which the impact function $f(\cdot)$ is concave, where a unique equilibrium is readily established. We then study convex impact functions and show that the contest may still yield a unique symmetric equilibrium.

2.1.1 Concave impact functions

Concave impact functions provide a stronger condition than weak log-concavity. It is well known that a concave impact function $f(\cdot)$ is sufficient for the existence and uniqueness of symmetric equilibria in a standard contest. We show that this condition guarantees the existence and uniqueness of symmetric equilibria in our context regardless of the prevailing disclosure policy.

Contest with disclosure We first consider the subgame where the contest organizer commits to the policy D. All contestants learn of N before they decide on their effort level. Each contestant i then rationally chooses his effort x_i to maximize the expected payoff

$$\pi_i = p_i(x_i, \mathbf{x}_{-i}; N)v - x_i. \tag{2}$$

Consider a subgame where N contestants participate. We now solve for the symmetric equilibrium of the contest. Define $H(x) \equiv \frac{f(x)}{f'(x)}$. As shown below, the equilibrium behavior of each contestant is characterized by the function $H(\cdot)$ and its inverse. $H(\cdot)$ is thus named as the *characteristic function* of the contest for convenience.

The symmetric equilibrium effort x is determined by the first order condition

$$H(x) = \frac{N-1}{N^2}v. \tag{3}$$

Because $f(x)$ is concave, we have $H'(x)>0$. As $H(0)=0$, there exists a unique $x>0$ which solves (3). The solution to (3) constitutes a unique symmetric pure-strategy equilibrium, if and only if it globally maximizes a representative contestant i's expected payoff π_i given that all others exert the same effort. We now formally establish the existence and uniqueness of a symmetric pure-strategy equilibrium.

Proposition 1 *Suppose that N (≥ 2) contestants participate in the contest. If they learn the actual number (N) of participants, each contestant in the unique symmetric pure-strategy equilibrium makes an effort*

$$x(N) = H^{-1}\left(\frac{N-1}{N^2}v\right) > 0, \tag{4}$$

where $H^{-1}(\cdot)$ is the inverse of the characteristic function $H(\cdot)$. The overall effort of the N-person contest is then given by

$$E(N) \equiv Nx(N) = N \cdot H^{-1}\left(\frac{N-1}{N^2}v\right). \tag{5}$$

Proof $x(N)$ of (4) is derived from the first order condition (3). To establish it as a symmetric equilibrium, it suffices to show that a representative contestant i's expected payoff π_i is globally concave in x_i given that all others exert the effort of (4). We have $\frac{\partial \pi_i}{\partial x_i} = \frac{(N-1)x(N)f'(x)}{[f(x)+(N-1)x(N)]^2}v - 1$. As $f'(x) \geq 0$ and $f''(x) \leq 0$, $f(x)$ increases and $f'(x)$ decreases with their arguments. Hence, $\frac{\partial \pi_i}{\partial x_i}$ decreases with x_i, i.e. π_i is concave in x_i: π_i increases with x_i when $x_i \leq x(N)$ and π_i decreases with x_i when $x_i \geq x(N)$. A symmetric equilibrium is therefore established where every contestant exerts effort $x(N)$. As (3) has a unique solution, the symmetric equilibrium with $x(N)$ is unique. □

Having obtained the solution to every possible contest with N participants, we are now ready to find the expected total effort of the game when the D-policy is adopted. Given the fixed entry probability q, the probability of $N \in \{0, 1, 2, \ldots, M\}$ contestants showing up is given by $\Pr(N) = C_M^N q^N (1-q)^{M-N}$. Hence, the expected total effort is given by

$$\begin{aligned} TE_D(q) &= \sum_{N=1}^{M} C_M^N q^N (1-q)^{M-N} N x(N) \\ &= \sum_{N=1}^{M} C_M^N q^N (1-q)^{M-N} N H^{-1}\left[\frac{1}{N}\left(1-\frac{1}{N}\right)v\right] \\ &= Mq \sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1} (1-q)^{M-N} H^{-1}\left[\frac{1}{N}\left(1-\frac{1}{N}\right)v\right]. \end{aligned} \tag{6}$$

Contest with concealment We now analyze the subgame in which the actual number of participants is not revealed by the contest organizer. A participant i chooses his effort x_i to maximize the expected payoff

$$\pi_i(x_i, \mathbf{x}_{-i}; q) = \sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1} (1-q)^{M-N} p_i(x_i, \mathbf{x}_{-i}; N)v - x_i.$$

Proposition 2 *If the actual number of participating contestants is not disclosed, each participant exerts an effort*

$$x_C(q) = H^{-1}\left[\sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1} (1-q)^{M-N} \frac{1}{N}\left(1-\frac{1}{N}\right)v\right], \tag{7}$$

in the unique symmetric pure-strategy equilibrium, where $H^{-1}(\cdot)$ *is the inverse of the characteristic function* $H(\cdot)$.

Proof We first assume that a symmetric equilibrium exists. The first order condition for effort is given by

$$\sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1}(1-q)^{M-N} \frac{N-1}{N^2} \frac{f'(x)}{f(x)} v - 1 = 0. \tag{8}$$

A concave $f(\cdot)$ implies that $\frac{f'(x)}{f(x)}$ must be monotonic. Hence, there exists a unique solution to the function, as given by (7). It remains to verify that $x_C(q)$ constitutes an equilibrium. First, note that $p_i(x_i, \mathbf{x}_{-i}; N)$ is concave. $\frac{d^2 p_i(x_i, \mathbf{x}_{-i}; N)}{dx_i^2} = \frac{f''(x_i)[f(x_i)+\sum_{j\neq i} f(x_j)]-2[f'(x_i)]^2}{[f(x_i)+\sum_{j\neq i} f(x_j)]^3} \sum_{j\neq i} f(x_j)$ is negative because of the concavity of $f(\cdot)$. Second, $\pi_i(x_i, \mathbf{x}_{-i}; q)$ is a weighted sum of $p_i(x_i, \mathbf{x}_{-i}; N)$. Hence, $\pi_i(x_i, \mathbf{x}_{-i}; q)$ must be concave in x_i as well. The global concavity ensures that the solution of (7) constitutes an equilibrium. □

Proposition 2 establishes the unique pure-strategy symmetric equilibrium of the contest with concealment. The expected overall effort in the subgame is therefore obtained as

$$\begin{aligned} TE_C(q) &= Mqx_C(q) \\ &= MqH^{-1}\left[\sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1}(1-q)^{M-N} \frac{1}{N}\left(1-\frac{1}{N}\right)v\right]. \end{aligned} \tag{9}$$

2.1.2 Convex impact functions

Symmetric equilibria in a contest do not necessarily require a concave impact function. However, a convex impact function would substantially complicate the analysis, because a contestant's payoff function may not be globally concave. In a two-player setting, Baye et al. (1994) have demonstrated the difficulty in characterizing the equilibria when the impact function becomes excessively convex. The analysis in our context can be further complicated by stochastic entries. Irregular problems could result, especially when N is concealed. In this case, each participant's payoff function $\pi_i(x_i, \mathbf{x}_{-i}; q)$ is the weighted sum of a set of non-monotonic functions with varying curvatures. More rigorous approaches are required to establish the equilibria in such games.

We now explore the possible equilibria when convex impact functions are in place. Two examples are discussed in order to demonstrate these possibilities. Because of the log-concavity of $f(x)$, (3) and (8) would continue to yield a unique solution, as given by (4) and (7), respectively. However, the solutions to first order conditions do not necessarily constitute an equilibrium. In the two examples discussed below, unique symmetric equilibria do exist and the results established in the previous section (Propositions 1 and 2, (6) and (9)) continue to apply.

We first consider the often-studied Tullock contest with impact function $f(x) = x^r$. The following can be obtained.

Claim 1 *When* $r \in (1, 1 + \frac{1}{M-1}]$, *there always exists a unique symmetric pure-strategy equilibrium.*

(a) *When N is disclosed, in a N-person contest, each participant exerts an effort* $x(N) = \frac{r(N-1)}{N^2}v > 0$.

(b) *When N is concealed, each participant exerts an effort* $x_C(q) = r\sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1}(1-q)^{M-N}\frac{1}{N}(1-\frac{1}{N})v > 0$.

Proof See Appendix. □

It is well known that when N, the number of participants, is common knowledge, a symmetric equilibrium exists in a Tullock contest if and only if $r \leq 1 + \frac{1}{N-1}$. When r falls below the cutoff $1 + \frac{1}{M-1}$, a unique symmetric equilibrium results in a contest with disclosure regardless of the actual number N. We further show that the cutoff also guarantees the existence of a unique symmetric equilibrium in a contest with concealment. The equilibrium effort outlays are adapted from (4) and (7), respectively. The overall effort in contests with disclosure and concealment can also be obtained from (6) and (9), respectively.

Further, we consider another family of convex impact functions that could also yield a symmetric equilibrium. Consider the family of impact functions $f(x) = e^{\alpha x} - 1$, with $\alpha \in (0, 1]$. For analytical convenience, the prize is normalized to $v = 1$. The following is then shown.

Claim 2 *Let $f(x) = e^{\alpha x} - 1$, with $\alpha \in (0, 1]$. When $M \leq 4$, a unique symmetric equilibrium exists in the contest regardless of the prevailing disclosure policy.*

(a) *When N is disclosed, in a N-person contest, each participant exerts the equilibrium effort*

$$x(N) = -\frac{1}{\alpha}\ln\left(1 - \frac{N-1}{N^2}\alpha\right). \tag{10}$$

(b) *When N is concealed, each participant exerts the equilibrium effort*

$$x_C(q) = -\frac{1}{\alpha}\ln\left[1 - \alpha\sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1}(1-q)^{M-N}\frac{N-1}{N^2}\right]. \tag{11}$$

Proof See Appendix. □

Claim 2 identifies another possible context in which a convex impact function renders symmetric equilibria. Again, (10) and (11) are adapted from (4) and (7) respectively. The overall effort in contests with disclosure and concealment can be obtained from (6) and (9), respectively.

2.2 Optimal disclosure policy

We now compare (6) with (9) to investigate the effort-maximizing disclosure policy. One can conclude that $TE_D(q) > TE_C(q)$, i.e. $\sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1}(1-q)^{M-N} H^{-1}[\frac{1}{N}(1-\frac{1}{N})v] > H^{-1}[\sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1}(1-q)^{M-N}\frac{1}{N}(1-\frac{1}{N})v]$, simply requires $H^{-1}(\cdot)$ to be convex by Jensen's Inequality, and therefore the characteristic function $H(\cdot) \equiv \frac{f(\cdot)}{f'(\cdot)}$ to be strictly concave. We summarize our results as follows.

Theorem 1 *Suppose that every contestant independently enters the contest with the same exogenous probability q and symmetric equilibria exist for contests with disclosure and concealment of number of entrants.*

(a) *Disclosing the actual number of contestants elicits strictly more (less) effort than concealing the actual number of contestants, if the characteristic function $H(\cdot)$ is strictly concave (convex).*
(b) *(Disclosure Irrelevance) The resultant expected total effort is independent of the disclosure policy, if the characteristic function $H(\cdot)$ is linear.*

We do not lay out a dedicated proof, but briefly interpret the logic that underpins our main result. Note that the function $H^{-1}(\cdot)$ (as well as its inverse $H(\cdot)$) plays a pivotal role in determining the equilibrium effort of each participating contestant. As revealed by (3) and (4), each contestant's equilibrium effort depends crucially on the properties of the characteristic function (and those of its inverse), which are fundamentally determined by the contest technology $f(\cdot)$. Recall from (4) that a contestant exerts an equilibrium effort $x(N) = H^{-1}(\frac{N-1}{N}v)$. The function $H(\cdot)$ thus depicts how contestants respond to the competitive environment of the contest, e.g., how they respond to changes in the number of competitors and/or the value of prize, etc. A given contest environment would trigger sharply different responses by contestants when the prevailing contest technologies (i.e., the characteristic functions) differ.

When N is to be concealed, each participating contestant exerts a uniform equilibrium effort $x_C(q) = H^{-1}[\sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1}(1-q)^{M-N}\frac{1}{N}(1-\frac{1}{N})v]$ upon entry. By way of contrast, when N is to be disclosed, each participating contestant responds to each realization of N by exerting an effort $x(N) = H^{-1}(\frac{N-1}{N^2}v)$ upon entry. On average, he exerts an effort $\sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1}(1-q)^{M-N} H^{-1}[\frac{1}{N}(1-\frac{1}{N})v]$.

A larger N implies that a less favorable contest is realized. Hence, when N is disclosed, a contestant exerts more effort when N (≥ 2) is small, while he exerts less effort when N is large.[3] A concave $H(\cdot)$ (i.e., a convex $H^{-1}(\cdot)$) implies that a contestant's equilibrium effort is increasingly elastic with respect to the value of its argument. A contestant tends to respond increasingly sensitively to any given decrease in N (by increasing effort $x(N)$), but less sensitively to any given increase in N. A strictly concave characteristic function leads a contestant to behave as if he were risk-loving when he supplies his effort, in spite of his risk-neutrality: a smaller N (a more favorable contest) incentivizes a contestant more than a larger N (a less favorable contest) disincentivizes him. Consequently, each contestant, on average, exerts more effort when N is disclosed than when it is concealed.

By way of contrast, when $H(\cdot)$ is convex (i.e., $H^{-1}(\cdot)$ is concave) and the realized N is disclosed, a contestant responds more sensitively to an increase in N (by lowering his effort), but less sensitively to a decrease in N. A strictly convex characteristic function leads a contestant to behave as if he were risk-averse: A larger N (a less favorable contest) disincentivizes him more than a smaller N (a more favorable contest) incentivizes him. This leads to the result that his overall expected effort $\sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1}(1-q)^{M-N} H^{-1}[\frac{1}{N}(1-\frac{1}{N})v]$ falls below $x_C(q)$.

Theorem 1(b) shows that $TE_D(q) = TE_C(q)$ if $H(x)$ is linear in x. Lim and Matros (2010) establish the "disclosure-irrelevance" principle in a Tullock contest with $f(x) = ax^r$. It can be directly verified that a linear characteristic function results if and only if a Tullock contest prevails. Note $H(0) = 0$ and $H'(\cdot) > 0$. Therefore, we must have $H(x) = tx$ if $H(\cdot)$ is a linear function, with constant $t > 0$. According to the definition of $H(x)$, we have $\frac{f'(x)}{f(x)} = \frac{1}{tx}$. Solving the differential equation yields $\ln f(x) = \ln(x^{\frac{1}{t}}) + b$, where b is a constant. It further leads to $f(x) = e^b x^{\frac{1}{t}}$, which takes the form of a power function.

[3] Note that $\frac{1}{N}(1-\frac{1}{N})$ decreases with N.

Our result reveals that the "disclosure-irrelevance" principle of Lim and Matros (2010) is essentially underpinned by the linearity of characteristics function $H(\cdot)$ that is associated with a "Tullock" contest.

There are many possible forms of $f(\cdot)$ that guarantee the existence of symmetric equilibria and lead to strictly concave or strictly convex characteristic functions. We present below two examples to illustrate these possibilities.

Example 1 Consider the family of functions $f(x) = [\ln(1+x)]^{\alpha}$, with $\alpha \in (0, 1]$. Simple calculus verifies $H(x) = \frac{f(x)}{f'(x)} = \alpha^{-1}(1+x)\ln(1+x)$, which further leads to $H'(x) = \alpha^{-1}[1+\ln(1+x)] > 0$ and $H''(x) = \frac{1}{\alpha}\frac{1}{1+x} > 0$. We then conclude that this functional form leads to a convex characteristic function.

Example 2 Consider the family of functions in Claim 2 of Sect. 2.1.2 $f(x) = e^{\alpha x} - 1$, with $\alpha \in (0, 1]$. As has been shown there, $H''(x) = -\alpha e^{-\alpha x} < 0$. This functional form then yields a concave characteristic function.

3 Extensions and discussion

This part of the paper further explores the issue of information disclosure from two additional dimensions. First, an extension that generalizes the disclosure policy in the basic setting by allowing the contest organizer to partially reveal the information on the actual number of participants is considered. Second, the commitment issue of disclosure policy is explored.

3.1 Imperfect information disclosure

We have assumed that the organizer of the contest either fully discloses the number of participating contestants, or completely withholds this information. We now allow the organizer to partially disclose her information.

Let the organizer's information disclosure strategy be depicted by an ordered set $(k_1, k_2, \ldots, k_I)$, where $k_i \in \{1, \ldots, M\}$ and $1 \leq I \leq M$. We arrange k_is in ascending order and let $k_I = M$. Each $(k_1, k_2, \ldots, k_I)$ thus characterizes a partition of the information space $\{1, 2, \ldots, M\}$. The organizer does not announce the exact realization of N, but discloses that N is in a partition set $\Omega_i = \{k_{i-1}+1, \ldots, k_i\}$, i.e., $k_{i-1}+1 \leq N \leq k_i$. For convenience, we assume $k_0 \equiv 0$.

When $I = M$, the finest partition is obtained. The partition strategy converges to a full disclosure strategy and the exact realization of N is revealed. When $I = 1$, the partition strategy is the coarsest, reducing to a concealment policy. The finer the partition, the more information on the actual number of contestants is revealed to contestants. We now investigate the optimal partition strategy of the organizer.

Define $P_i = \sum_{t=k_{i-1}}^{k_i-1} C_{M-1}^t q^t (1-q)^{(M-1)-t}$, $i = 1, 2, \ldots, I$. P_i is the conditional probability that a participant faces a competition where the total number of contestants falls within the range Ω_i.

When a contestant participates in the contest and is informed that $N \in \Omega_i$, he has to form a posterior belief of the number of competitors. He will be competing against t contestants with a probability $\Pr(t|\Omega_i) = \frac{C_{M-1}^t q^t (1-q)^{(M-1)-t}}{P_i}$, $t = k_{i-1}, \ldots, k_i - 1$. Similar to (7), an entrant would exert an effort $x(\Omega_i) = H^{-1}(\sum_{t=k_{i-1}+1}^{k_i} \Pr(t-1|\Omega_i)\frac{1}{t}(1-\frac{1}{t})v)$, which is obtained from the first order condition. It can be established as a global optimum through the use of techniques that are utilized in Sect. 2.1.1 when $f(x)$ is concave, or those utilized in Sect. 2.1.2 when $f(x)$ belongs to the two families of convex impact functions discussed there. These proofs are not repeated for the sake of brevity.

We can immediately obtain that each participant on average expends an expected effort

$$Ex = \sum_{i=1}^{I} P_i x(\Omega_i).$$

We then conclude the following.

Theorem 2 *Suppose that every contestant independently enters the contest with the same exogenous probability q, and symmetric equilibria exist for contests with disclosure and concealment of the partition sets.*

(a) *If the characteristic function $H(\cdot)$ is strictly concave* (*strictly convex*), *the contest organizer fully discloses* (*fully conceals*) *the actual number of participating contestants, and partial disclosure is never optimal.*
(b) (Disclosure Irrelevance) *The resultant expected total effort is independent of the disclosure strategy* (*i.e., how the partitions are constructed*), *if the characteristic function $H(\cdot)$ is linear, where a Tullock contest with $f(x) = ax^r$ applies.*

Proof Let us merge two arbitrary neighbor partition sets Ω_j and Ω_{j+1}. After the merger, we denote $\tilde{\Omega} = \Omega_j \cup \Omega_{j+1}$. Define $\tilde{P} = \sum_{t=k_{j-1}}^{k_{j+1}-1} C_{M-1}^t q^t (1-q)^{(M-1)-t} = P_j + P_{j+1}$. Then $\tilde{P}$ is the conditional probability that a participant would face a competition where the total number of contestants falls in $\tilde{\Omega}$. The expected effort of an entrant is given by

$$\tilde{E}x = \tilde{P}x(\tilde{\Omega}) + \sum_{i \neq j, j+1} P_i x(\Omega_i).$$

To compare Ex and $\tilde{E}x$, we only need to compare $\sum_{i=j}^{j+1} P_i x(\Omega_i)$ with $\tilde{P}x(\tilde{\Omega})$. Note that

$$\tilde{P}x(\tilde{\Omega}) = (P_j + P_{j+1}) H^{-1}\left(\sum_{t=k_{j-1}+1}^{k_{j+1}} \frac{C_{M-1}^{t-1} q^{t-1} (1-q)^{M-t}}{P_j + P_{j+1}} \frac{1}{t}\left(1 - \frac{1}{t}\right) v \right),$$

$$\text{and} \quad P_j x(\Omega_j) = P_j H^{-1}\left(\sum_{t=k_{j-1}+1}^{k_j} \frac{C_{M-1}^{t-1} q^{t-1} (1-q)^{M-t}}{P_j} \frac{1}{t}\left(1 - \frac{1}{t}\right) v \right).$$

If the $H^{-1}(\cdot)$ is strictly concave, i.e., the characteristic function $H(\cdot)$ is strictly convex, then

$$\sum_{i=j}^{j+1} P_i x(\Omega_i)$$

$$= (P_j + P_{j+1}) \sum_{i=j}^{j+1} \frac{P_i}{P_j + P_{j+1}} x(\Omega_i)$$

$$\le (P_j + P_{j+1}) H^{-1}\left[\sum_{i=j}^{j+1}\left(\frac{P_i}{P_j + P_{j+1}} \sum_{t=k_{i-1}+1}^{k_i} \frac{C_{M-1}^{t-1} q^{t-1}(1-q)^{M-t}}{P_i} \frac{1}{t}\left(1-\frac{1}{t}\right)v\right)\right]$$

$$= (P_j + P_{j+1}) H^{-1}\left[\sum_{t=k_{j-1}+1}^{k_{j+1}} \frac{C_{M-1}^{t-1} q^{t-1}(1-q)^{M-t}}{P_j + P_{j+1}} \frac{1}{t}\left(1-\frac{1}{t}\right)v)\right]$$

$$= \tilde{P} x(\tilde{\Omega}).$$

In this case, a coarser partition strategy always leads to more effort. At the optimum, the organizer creates only one partition set ($I = 1$ and $k_1 = M$), i.e., she discloses no information to participating contestants.

When the characteristic function is strictly concave, the comparison is reversed: the finer the partition strategy, the more effort is expended in the contest. The optimum requires full information disclosure, i.e., $I = M$.

When the characteristic function is linear, where a Tullock contest applies and $f(x)$ takes the form $f(x) = ax^r$, merging the two partitions does not affect equilibrium effort.

We then obtain the results of Theorem 2. □

Theorem 2 strengthens the argument of Theorem 1. The results of Theorem 1 are robust even when a partial disclosure strategy is allowed in the game. It further verifies that the optimal disclosure policy depends crucially on the concavity of the characteristic function. More importantly, partial disclosure never emerges in the equilibrium if the characteristic function is strictly concave or strictly convex.

We again find that the "disclosure irrelevance" principle applies in the case of linear characteristic functions (i.e., Tullock contests). Theorem 2(b) substantially adds to our knowledge of behavior in this type of contest: the equilibrium level of effort expended in the contest does not depend on whether the contest organizer discloses information and how much information is disclosed, despite there being numerous ways to construct a partition disclosure strategy!

3.2 Commitment of disclosure policy

We assume that the contest organizer commits to her disclosure policy prior to the realization of the actual number of contestants. We follow the standard literature on mechanism design, such as Myerson (1981), and assume that the contest organizer has commitment power. Lim and Matros (2010) have also studied a case where the organizer is unable to commit, and can decide whether or not to disclose the actual number of participants after the number has been realized. They showed that the contest organizer would be unable to conceal the information, and she always reveals it in equilibrium. The same result would be obtained in the setting studied in this paper, regardless of the contest technology.[4]

It should be noted that the inability to commit could harm the contest organizer, as it has been shown here that concealing the actual number of contestants can elicit more effort, when the characteristic function $H(\cdot)$ is convex. Hence, it would be theoretically interesting and important to explore the mechanisms that strengthen the commitment power of the contest organizer. A thorough analysis on the commitment issue of disclosure policy is beyond

[4] A detailed proof is omitted for the sake of brevity, but it is available from the authors upon request.

the scope of this study, but will be pursued by the authors in future studies. However, two remarks are in order to address this issue.

First, the contest organizer can seek third parties to maintain the credibility of her disclosure policy. One mechanism for this is to resort to obtaining certification from the relevant authorities, such as notaries, to verify the integrity of the committed contest rules. When the characteristic function is convex, it would be incentive-compatible to exercise such a procedure in order to maintain a concealment policy in the contest, provided it does not entail prohibitive certifying costs. Alternatively, the contest organizer may outsource or delegate the administrative task to independent parties, which carry out the rules of the contest on her behalf.

Second, the contest organizer can carry out a concealment policy more credibly when she sponsors the contest not once but repeatedly over time. Insights can be borrowed from the notion of "reputation equilibria", and the extensive literature on reputation building.[5] Reputation concerns create a trade-off between immediate gains and long-run payoffs, and provide the contest organizer with additional incentives to maintain her concealment policy. Although the contest organizer can be tempted to reveal the actual number of contestants when it turns out to be low (which, if revealed, would incentivize each participant to supply more effort) in a single contest, she may refrain from doing so since it prevents her from establishing her reputation, and the loss can outweigh the temporary advantage. Deviation in one period changes the beliefs of the contestants. By a logic analogous to the full-revelation result in single-period contests (see Lim and Matros 2010), the organizer may have to reveal the information in all future periods. This necessarily leads to less future effort on average.

4 Concluding remarks

The current study examines the impact of disclosure on expected effort in contests with a stochastic number of contestants. Our analysis provides important insights into the design of a contest with a stochastic number of contestants. We showed that whenever the characteristic function $H(x) = \frac{f(x)}{f'(x)}$ is linear (i.e., Tullock contest technology), the expected total effort in a contest does not depend on how much information on the actual number of contestants is revealed to participants. However, this result does not hold when the characteristic function is nonlinear. The comparison is determined by the concavity of the characteristic function.

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Appendix: Proofs

Proof of Claim 1 When N is disclosed, it is well known that a unique pure-strategy symmetric equilibrium exists, and the solution is not different from (4). The analysis is less explicit

[5] Reader is referred to Shapiro (1982, 1983), Fudenberg and Levin (1989), Fudenberg et al. (1990), and Kreps (1990).

in the case where N is concealed. We then examine the payoff function $\pi_i(x_i, \mathbf{x}_{-i}; q)$. (7) still solves (8), but it has yet to be established as a global maximizer of $\pi_i(x_i, \mathbf{x}_{-i}; q)$ given that all other participants exert the same effort.

One can verify $\xi_N(x_i) = \frac{\partial^2 p_i(x_i,\mathbf{x}_{-i};N)}{\partial x_i^2}|_{x_{-i}=x_C(q)} = \frac{-(r+1)x_i^r+(r-1)(N-1)(x_C(q))^r}{[x_i^r+(N-1)(x_C(q))^r]^3} r x_i^{r-2}(N-1)(x_C(q))^r$. Thus $\xi_N(x_i)$ is positive if $x_i^r < \frac{r-1}{r+1}(N-1)(x_C(q))^r$, and negative if $x_i^r > \frac{r-1}{r+1}(N-1)(x_C(q))^r$. This implies that $\Phi_N(x_i) = \frac{\partial p_i(x_i,\mathbf{x}_{-i};N)}{\partial x_i}|_{x_{-i}=x_C(q)}$ is not monotonic: It is increasing if $x_i^r < \frac{r-1}{r+1}(N-1)(x_C(q))^r$, and decreasing if $x_i^r > \frac{r-1}{r+1}(N-1)(x_C(q))^r$. Clearly $\frac{r-1}{r+1}(N-1) \le 1$ if and only if $r \le \frac{N}{N-2}$. Because $r \le 1 + \frac{1}{M-1}$, we must have $\frac{r-1}{r+1}(N-1) < 1$ for all $N \le M$.

Let $\Phi(x_i) = \sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1}(1-q)^{M-N} \frac{\partial p_i(x_i,\mathbf{x}_{-i};N)}{\partial x_i}|_{x_{-i}=x_C(q)}$, and $\xi(x_i) = \sum_{N=1}^{M} C_{M-1}^{N-1} q^{N-1}(1-q)^{M-N} \frac{\partial^2 p_i(x_i,\mathbf{x}_{-i};N)}{\partial x_i^2}|_{x_{-i}=x_C(q)}$. Since $\frac{r-1}{r+1}(N-1) < 1$ for all $N \le M$, we have $x_i^r > \frac{r-1}{r+1}(N-1)(x_C(q))^r$ when $x_i = x_C(q)$ for all $N \le M$, which means $\xi(x_i)|_{x_i=x_C(q)} < 0$. This leads to $\frac{d^2\pi_i(x_i,\mathbf{x}_{-i};q)}{dx_i^2}|_{x_i=x_{-i}=x_C(q)} = v\xi(x_i)|_{x_i=x_C(q)} < 0$. Hence, $x_i = x_C(q)$ must be at least a local maximizer of $\pi_i(x_i, \mathbf{x}_{-i}; q)$ when $x_{-i} = x_C(q)$.

When $x_i < [\frac{r-1}{r+1}]^{1/r} x_C(q)$, $\xi_N(x_i) > 0$ for all $N \le M$. Hence, we have $\xi(x_i) > 0$ when $x_i < [\frac{r-1}{r+1}]^{1/r} x_C(q)$, which means that $\Phi(x_i)$ increases when $x_i < [\frac{r-1}{r+1}]^{1/r} x_C(q)$. Similarly, $\xi(x_i) < 0$ when $x_i > [\frac{r-1}{r+1}(M-1)]^{1/r} x_C(q)$, which means that $\Phi(x_i)$ decreases when $x_i > [\frac{r-1}{r+1}(M-1)]^{1/r} x_C(q)$. We next show that there exists a unique $x' \in ([\frac{r-1}{r+1}]^{1/r} x_C(q), [\frac{r-1}{r+1}(M-1)]^{1/r} x_C(q))$ such that $\Phi(x_i)$ increases (decreases) if and only if $x_i < (>)\ x'$. For this purpose, it suffices to show that there exists a unique $x' \in ([\frac{r-1}{r+1}]^{1/r} x_C(q), [\frac{r-1}{r+1}(M-1)]^{1/r} x_C(q))$, such that $\xi(x') = 0$.

First, such x' must exist by continuity of $\xi(x_i)$. As have been revealed, $\xi(x_i) > 0$ when $x_i < [\frac{r-1}{r+1}]^{1/r} x_C(q)$; and $\xi(x_i) < 0$ when $x_i < [\frac{r-1}{r+1}(M-1)]^{1/r} x_C(q)$.

Second, the uniqueness of such x' can be verified as below. We have

$$\begin{aligned}
&\left.\frac{\partial^3 p_i(x_i, \mathbf{x}_{-i}; N)}{\partial x_i^3}\right|_{x_{-i}=x_C(q)} \\
&\quad = r(N-1)(x_C(q))^r \Bigg\{ (r-2) x_i^{r-3} \frac{-(r+1)x_i^r + (r-1)(N-1)(x_C(q))^r}{[x_i^r + (N-1)(x_C(q))^r]^3} \\
&\qquad + x_i^{r-2} \frac{-r(r+1)x_i^{r-1}[x_i^r + (N-1)(x_C(q))^r] - 3r x_i^{r-1}[-(r+1)x_i^r + (r-1)(N-1)(x_C(q))^r]}{[x_i^r + (N-1)(x_C(q))^r]^4} \Bigg\} \\
&\quad = \frac{r(N-1)(x_C(q))^r x_i^{r-3}}{[x_i^r + (N-1)(x_C(q))^r]^3} \Bigg\{ (r-2)[-(r+1)x_i^r + (r-1)(N-1)(x_C(q))^r] \\
&\qquad + \frac{-r(r+1)x_i^r[x_i^r + (N-1)(x_C(q))^r] - 3r x_i^r[-(r+1)x_i^r + (r-1)(N-1)(x_C(q))^r]}{[x_i^r + (N-1)(x_C(q))^r]} \Bigg\} \\
&\quad = \frac{r(N-1)(x_C(q))^r x_i^{r-3}}{[x_i^r + (N-1)(x_C(q))^r]^3} \Bigg\{ (r-2)[-(r+1)x_i^r + (r-1)(N-1)(x_C(q))^r] \\
&\qquad + \frac{2r x_i^r}{[x_i^r + (N-1)(x_C(q))^r]} [(r+1)x_i^r - (2r-1)(N-1)(x_C(q))^r] \Bigg\}.
\end{aligned}$$

Recall $\xi_N(x_i) = \frac{-(r+1)x_i^r+(r-1)(N-1)(x_C(q))^r}{[x_i^r+(N-1)(x_C(q))^r]^3} r x_i^{r-2}(N-1)(x_C(q))^r$. We then have

$$\begin{aligned}&\left.\frac{\partial^3 p_i(x_i, \mathbf{x}_{-i}; N)}{\partial x_i^3}\right|_{x_{-i}=x_C(q)}\\&= (r-2)x_i^{-1}\xi_N(x_i)\\&\quad+\frac{2r^2(N-1)(x_C(q))^r x_i^{2r-3}}{[x_i^r+(N-1)(x_C(q))^r]^4}[(r+1)x_i^r-(2r-1)(N-1)(x_C(q))^r].\end{aligned}$$

We now claim $[(r+1)x_i^r-(2r-1)(N-1)(x_C(q))^r]$ is negative for all $x_i \le [\frac{r-1}{r+1}(M-1)]^{1/r}x_C(q)$. A detailed proof is as follows. From $x_i \le [\frac{r-1}{r+1}(M-1)]^{1/r}x_C(q)$, we have $(r+1)x_i^r \le (r-1)(M-1)(x_C(q))^r$. To show $(r+1)x_i^r-(2r-1)(N-1)(x_C(q))^r < 0$, it suffices to show $(r-1)(M-1) < (2r-1)(N-1)$ when $N=2$, which requires $r < 1+\frac{1}{M-3}$. This holds as $r < 1+\frac{1}{M-1}$.

We now show that at any $x_i \in ([\frac{r-1}{r+1}]^{1/r}x_C(q), [\frac{r-1}{r+1}(M-1)]^{1/r}x_C(q))$ such that $\xi(x_i)=0$, $\xi(x_i)$ must be locally decreasing. Note $\frac{\partial^3 p_i(x_i,\mathbf{x}_{-i};N)}{\partial x_i^3}|_{x_{-i}=x_C(q)} = (r-2)x_i^{-1}\xi_N(x_i)+A_N(x_i)$ where $A_N(x_i) = \frac{2r^2(N-1)(x_C(q))^r x_i^{2r-3}}{[x_i^r+(N-1)(x_C(q))^r]^4}[(r+1)x_i^r-(2r-1)(N-1)(x_C(q))^r] < 0$. Thus

$$\begin{aligned}\frac{\partial\xi(x_i)}{\partial x_i} &= (r-2)x_i^{-1}\sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}\xi_N(x_i)+\sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}A_N(x_i)\\&= (r-2)x_i^{-1}\xi(x_i)+\sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}A_N(x_i)\\&= \sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}A_N(x_i) < 0.\end{aligned}$$

We are now ready to show the uniqueness of x' by contradiction. Suppose that there exists more than one zero points x' and x'' for $\xi(x_i)$ with $x' \neq x''$. Because $\xi(x_i)$ must be locally decreasing, then there must exist at least another zero point $x''' \in (x', x'')$ at which $\xi(x_i)$ must be locally increasing. This cannot be true. Contradiction thus results. Hence, such a zero point x' of $\xi(x_i)$ must be unique.

Recall $\Phi(x_i)$ increases (decreases) if and only if $x_i < (>)\ x'$ and it reaches its maximum at x'. Note $\frac{\partial\pi_i(x_i,\mathbf{x}_{-i};q)}{\partial x_i}|_{x_{-i}=x_C(q)} = v\Phi(x_i)-1$ and $\Phi(0)=0$. $\frac{\partial\pi_i(x_i,\mathbf{x}_{-i};q)}{\partial x_i}|_{x_{-i}=x_C(q)}$ at most has two zero points. Note $x_i = x_C(q)$ must be a zero point for $\frac{\partial\pi_i(x_i,\mathbf{x}_{-i};q)}{\partial x_i}|_{x_{-i}=x_C(q)}$ by definition. One can further verify that $\pi_i(x_C(q),\mathbf{x}_{-i};q)|_{x_{-i}=x_C(q)} > \pi_i(0,\mathbf{x}_{-i};q)|_{x_{-i}=x_C(q)} = v(1-q)^{M-1}$ as follows. We have $\pi_i(x_C(q),\mathbf{x}_{-i};q)|_{x_{-i}=x_C(q)} = \sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}\frac{1}{N}v - x_C(q) = \sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}\frac{1}{N}v - r\sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}\frac{1}{N}(1-\frac{1}{N})v = v(1-q)^{M-1}+\sum_{N=2}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}[\frac{1}{N}-r\frac{1}{N}(1-\frac{1}{N})]v$. The terms $\frac{1}{N}-r\frac{1}{N}(1-\frac{1}{N})$, $N \ge 2$ are apparently positive because $r(1-\frac{1}{N}) \le \frac{M}{M-1} \times \frac{N-1}{N} \leqq 1$ if and only if $N \leqq M$.

Since $\pi_i(x_C(q),\mathbf{x}_{-i};q)|_{x_{-i}=x_C(q)} > \pi_i(0,\mathbf{x}_{-i};q)|_{x_{-i}=x_C(q)}$, $\frac{\partial\pi_i(x_i,\mathbf{x}_{-i};q)}{\partial x_i}|_{x_{-i}=x_C(q)}$ must have two zero points, and $x_i = x_C(q)$ is the local maximum point of $\pi_i(x_i,\mathbf{x}_{-i};q)|_{x_{-i}=x_C(q)}$ and the other is the local minimum point. Hence, $x_i = x_C(q)$ is the global best response. □

Proof of Claim 2 We first consider the contest with disclosure. When $N = 1$, the entrant clearly exerts zero effort. When $N \geq 2$, we claim that all entrants exert an equilibrium effort of $x(N) = H^{-1}(\frac{N-1}{N^2}) = -\frac{1}{\alpha}\ln(1 - \frac{N-1}{N^2}\alpha)$. To prove this claim, we need to show that when $x_j = x(N)$ for $j \neq i$, $x_i = x(N)$ maximizes $\pi_i = p_i(x_i, \mathbf{x}_{-i}; N)v - x_i = \frac{e^{\alpha x_i}-1}{(e^{\alpha x_i}-1)+(N-1)(e^{\alpha x(N)}-1)} - x_i$. $\frac{\partial \pi_i}{\partial x_i} = (N-1)(e^{\alpha x(N)} - 1)\alpha \frac{e^{\alpha x_i}}{[(e^{\alpha x_i}-1)+(N-1)(e^{\alpha x(N)}-1)]^2} - 1 = \frac{(\frac{N-1}{N})^2\alpha}{1-\frac{N-1}{N^2}\alpha}\alpha\frac{e^{\alpha x_i}}{[e^{\alpha x_i}-\Phi]^2} - 1$ with $\Phi = 1 - \frac{(\frac{N-1}{N})^2\alpha}{1-\frac{N-1}{N^2}\alpha} \in (0,1)$ as $\alpha \in (0,1]$. Hence, $\frac{\partial \pi_i}{\partial x_i}|_{x_i=0} = \frac{1-\frac{N-1}{N^2}\alpha}{(\frac{N-1}{N})^2} - 1 > 0$. Let $\Delta(y) = \frac{y}{[y-\Phi]^2}$, where $y \geq 1$. We have $\frac{d\Delta}{dy} = \frac{1}{[y-\Phi]^2} - 2\frac{y}{[y-\Phi]^3} = \frac{-1}{[y-\Phi]^3}[y+\Phi] < 0$, which implies that $\frac{\partial \pi_i}{\partial x_i}$ decreases with x_i. Hence, the solution of $x(N)$ from first order condition (3) is the unique global maximizer.

We now consider the contest with concealment. We will show that when $M \leq 4$, all entrants exert an equilibrium effort of $x_C(q)$ of (7), i.e.

$$x_C(q) = -\frac{1}{\alpha}\ln\left[1 - \alpha\sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}\frac{N-1}{N^2}\right].$$

Since $\frac{N-1}{N^2} = \frac{1}{N}(1 - \frac{1}{N})$ decreases with $N \geq 2$ and $\sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N} = 1$, $x_C(q) \leq x(2) = -\frac{1}{\alpha}\ln[1 - \frac{\alpha}{4}]$. Hence, $f(x_C(q)) = e^{\alpha x_C(q)} - 1 \leq e^{\alpha x(2)} - 1 = \frac{\frac{\alpha}{4}}{1-\frac{\alpha}{4}} \leq \frac{1}{3}$, because $\alpha \in (0,1]$. It further implies that $(N-1)f(x_C(q)) \leq 1$ as long as $N \leq M \leq 4$. We are now ready to show that when $x_j = x_C(q)$ for $j \neq i$, $x_i = x_C(q)$ maximizes $\pi_i(x_i, \mathbf{x}_{-i}; q) = \sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}p_i(x_i, \mathbf{x}_{-i}; N)v - x_i = \sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}\frac{e^{\alpha x_i}-1}{(e^{\alpha x_i}-1)+(N-1)(e^{\alpha x_C(q)}-1)} - x_i$. It suffices to show that $\pi_i(x_i, \mathbf{x}_{-i}; q)$ is concave in x_i. $\frac{\partial \pi_i(x_i,\mathbf{x}_{-i};q)}{\partial x_i} = \sum_{N=1}^{M} C_{M-1}^{N-1}q^{N-1}(1-q)^{M-N}\frac{\partial \Psi_N}{\partial x_i} - 1$, where $\Psi_N = \frac{e^{\alpha x_i}-1}{(e^{\alpha x_i}-1)+(N-1)(e^{\alpha x_C(q)}-1)}$. We have $\frac{\partial \Psi_N}{\partial x_i} = (N-1)(e^{\alpha x_C(q)} - 1)\alpha\frac{e^{\alpha x_i}}{[e^{\alpha x_i}-\Phi_N]^2}$, where $\Phi_N = 1 - (N-1)f(x_C(q)) = 1 - (N-1)(e^{\alpha x_C(q)} - 1) \in [0,1]$ since $N \leq M \leq 4$. Note that $\frac{e^{\alpha x_i}}{[e^{\alpha x_i}-\Phi_N]^2}$ decreases with x_i when $\Phi_N \in [0,1]$. The concavity of $\pi_i(x_i, \mathbf{x}_{-i}; q)$ with respect to x_i is thus guaranteed when $x_j = x_C(q)$ for $j \neq i$. Because $x_C(q) > 0$, $\pi_i(x_i, \mathbf{x}_{-i}; q)$ increases with x_i when $x_i \leq x_C(q)$ and $\pi_i(x_i, \mathbf{x}_{-i}; q)$ decreases with x_i when $x_i \geq x_C(q)$, which guarantees that the solution to (8) constitutes a symmetric equilibrium. The uniqueness of symmetric equilibrium is implied by the monotonicity of $H(\cdot)$. □

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