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ON ELIMINATION IN CONTESTS: A PERSPECTIVE ON OUTPUT MAXIMIZATION

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Abstract. This paper investigates the desirability of adding a preliminary elimination stage for output maximization in a winner-take-all contest framework in which the contestant who achieves the highest (random) output wins. We find that, generally, the desirability of an elimination stage does not monotonically depend on the productivity of the effort; adding a preliminary stage can improve output for both concave and convex production functions. This result contrasts sharply with current insight from effort maximization, which argues that adding a preliminary stage can increase effort supply only if the production function is concave.

1. INTRODUCTION

A contest is a situation in which economic entities expend costly effort to win a valuable prize. Contests have been extensively studied in a variety of contexts, such as rent seeking, lobbying, political campaigns, sports, R&D races, competitive procurement and college admissions. Contest design, which is the creation of the rules that define which contestants will be victorious, has been studied in a huge body of economic literature.[1] A growing strand of this literature has recognized multi-stage elimination contests in which there are preliminary stages and a final stage.[2] The contestants are successively eliminated from the race through earlier stages, and only the survivors in each stage compete against others to advance further in the competition.

Various examples are available to illustrate multi-stage elimination contests. For example, the International Olympic Commitee selected five cities (London, Madrid, Moscow, New York and Paris) out of nine as 'finalists' for the 2012 Summer Olympic Games. In the World Cup, teams are divided into different regional groups in preliminary competitions, and the winners of these competitions are chosen for the final competition (Amegashie, 1999; Stein and Rapoport, 2005). Similarly, when recruiting new faculty members, university departments interview a large number of candidates, but only a small subset can be invited for a campus visit (Fu and Lu, 2012). In the early stages of these scenarios, contestants mainly strive to avoid elimination.

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[1] See Gradstein (1998) for an overview of the literature.

[2] Multi-stage contests are studied by: Amegashie (1999, 2000, 2003); Baik and Kim (1997); Baik and Lee (2000); Baik and Shogren (1995); Clark and Riis (1996); Fu and Lu (2012); Gradstein (1998); Gradstein and Konrad (1999); Higgins *et al*. (1985); Katz and Tokatlidu (1996); Rosen (1986); and Stein and Rapoport (2005).

An important dimension of this aspect of elimination contests, which we call the 'shortlisting structure', involves whether and how to segment the contestant population, how to choose the number of stages, and how to distribute the prize money among a set of prizes of differing ranks in a multi-stage elimination contest. In the literature, this strand can be traced back to Rosen's seminal paper (1986), in which he searches for a reward scheme in a 2^N contestant N-stage sequential contest. Gradstein (1998) makes an initial attempt to conduct a comparative analysis of rent-seeking contests in terms of the amount and timing of effort they elicit from participants. Later, Amegashie (1999) examines the practice of selecting finalists in rent-seeking contests based on the rent-seekers' efforts in a preliminary competition. Amegashie (2000) also compares the two elimination procedures (pooling contestants and matching contestants) in a two-stage Tullock contest with linear contest technology, and concludes that pooling competition generates more effort. Fu and Lu (2012) investigate the optimal structure of a multi-stage sequential elimination contest. Similar to Amegashie (1999, 2000), Gradstein and Konrad (1999) and many others, the contest success function we use for the winner selection mechanism in each stage is Tullock's probabilistic model (Tullock, 1980).[3]

While contestants are assumed to take the prize as given, a contest organizer may wish to implement an optimal structure to achieve a given objective. An employer, for instance, may wish to maximize expected total output from all employees; a government with firms in a patent race or a firm with several R&D scientists may wish to maximize the performance of the winner; in sports, a team will seek to maximize the performance of all the contestants. Indeed, most work on contest design in the past decades has focused on the maximization of total effort by contenders. In a two-stage competition of Amegashie (1999), it is shown that if the discrimination power is either very high in both stages or, in the final stage, is sufficiently higher than the discrimination power in the preliminary stage, the two-stage design results in lower total effort than in the single-stage design. Gradstein and Konrad (1999) find that the simultaneous (single-stage) contest of all participants is the effort-maximizing contest structure when the discrimination power is greater than one, and the pairwise multistage contest is the effort-maximizing structure when the discrimination power is smaller than one.[4]

Because the decision-maker cannot reward contestants based on their effort (because this is neither observable nor verifiable, as pointed out in the moral hazard model), she ranks these contestants by their perceived output in descending order. That is, the higher the perceived output, the better a contestant's rank. With regard to a contest organizer's objective, a more realistic situation is that the decision-maker strictly prefers higher expected total output. As an employer

[3] The basic Tullock contest model measures the impact of a contestant's effort on the winning probabilities through a power function of his effort; that is, x^r. The parameter r is a measure of the discrimination of the selection mechanism.

[4] With discrimination exponents r greater (less) than or equal to one, the convex (concave) impact function then exhibits increasing (decreasing) return on effort.

who aims at maximizing profit with fixed production cost, for example, the decision-maker (referred to herein as 'she') is more concerned about the firm's overall production than about how much effort she can elicit from employees, because productivity is not always positively related to effort. A smart employee, for instance, can be very productive with a small effort. In a governmental procurement process for research proposals, the government is more interested in the overall scientific achievements of research institutes than in the effort level; one department may not succeed, even with a lot of effort, while another might be innovative with little effort.

Based on the above ideas, structural design that maximizes overall expected total output is the focal point of this paper. We study the optimal design of contests from the perspective of output maximization, focusing on two issues: (i) given the number of shortlists at the preliminary stage, how should the contest organizer design the group size; and (ii) comparing a two-stage contest with a single-stage contest, should the contest organizer add a preliminary elimination stage. Applying the framework of Amegashie (1999), our results indicate that in a two-stage contest with a preliminary stage, the optimal structure is each competing group with equivalent competitors. Given the total number of contestants and the number of finalists in the preliminary stage, the total output level does not monotonically depend on the productivity of the effort, r. Adding a preliminary stage can improve output for both concave and convex production functions. When the productivity level is so low that it converges to zero, adding an optimally-designed preliminary stage leads to more expected output, because the output level for each individual converges to one regardless of his or her effort. Moreover, with a sufficiently large number of contestants, when $0 < r < 1$, the single-stage contest emerges as the output-maximizing structure. However, we can always find a small right neighbourhood of $r = 1$ such that within this neighbourhood, the two-stage contest is the output-maximizing contest structure, because the convex (concave) production function transfers effort into output increasingly (decreasingly). These results absolutely contrast to the current insight on effort maximization, which indicates that adding a preliminary stage can never benefit expected effort when production function is convex.[5]

The paper proceeds as follows. In Section 2 we set up the model of output competition. Section 3 lays out the two-stage contest framework, elaborates on the optimal output-maximizing contest structure, and compares a two-stage contest with a single-stage pooling contest. Section 4 concludes.

2. A CONTEST MODEL OF OUTPUT COMPETITION

There are N (≥ 2) risk-netural contestants, who produce random outputs from their effort input, x_i, following a random production function as below:

[5] Amegashie (1999) and Gradstein and Konrad (1999) use different frameworks, while both of their results show that a preliminary stage could not improve the total effort when the impact function is convex.

$$y_i = x_i^r \varepsilon_i, \forall i. \quad (1)$$

In equation 1, x_i^r catches the deterministic part of the production process that is solely determined by the effort, x_i, while the noise term, ε_i, catches the randomness in the production process. The specification of equation 1 represents a standard setting of moral hazard in which contestants' effort, x_i, is not observable, while their output levels y_i are observable. We assume that the idiosyncratic noises $\boldsymbol{\varepsilon} \triangleq \{\varepsilon_i, i \in \mathrm{N}\}$ are independently and identically distributed with zero means.

While effort is not observable but output is observable, the incentive scheme that induces effort and output must be based on observable outputs. A winning rule that is based on the ranking of the outputs is natural: the best performer wins the contest. In the specification of equation 1, the best performer is the contestant who achieves the highest output, y_i. Many sports, as well as labour competitions, demonstrate this winning rule. A weightlifting competition honours the player who raises the heaviest successfully, and a company awards the branch that achieves the highest profit.

In this paper, we adopt such a winning rule and study how an additional elimination stage that shortlists finalists would affect the total output. Ties are broken randomly. We use **N** to denote the set of all contestants. Suppose $\boldsymbol{\Omega}$ is a subgroup of **N**. We use $\mathbf{x}_{\boldsymbol{\Omega}}$ to denote the effort vector of the contestants in group $\boldsymbol{\Omega}$.

Lemma 1. *For any given* $\mathbf{x}_{\boldsymbol{\Omega}} \geq 0$ *such that* $\sum_{j \in \Omega} x_j^r > 0$, *the likelihood that a contestant* $i \in \boldsymbol{\Omega}$ *wins the competition is:*

$$p(i|\mathbf{x}_{\boldsymbol{\Omega}}) = \frac{x_i^r}{\sum_{j \in \boldsymbol{\Omega}} x_j^r}, \forall i \in \boldsymbol{\Omega}, \forall \boldsymbol{\Omega} \in \mathbf{N}. \quad (2)$$

The proof is omitted but is available from McFadden (1973, 1974). The likelihood that a contestant $i \in \boldsymbol{\Omega}$ wins is simply the probability that his output is the highest among all contestants in group $\boldsymbol{\Omega}$ (the contestant is herein referred to as 'he'). This result shows that a generalized Tullock success function can be generated by designating the contestant with the highest output to be the winner.

The contest organizer has a total budget V, which can be used as a winner-take-all prize. In a single-stage winner-take-all contest, a representative contestant i maximizes his payoff of

$$\pi(x_i, \mathbf{x}_{-i}) = p(i|\mathbf{x}_{\mathbf{N}})V - x_i = \frac{x_i^r}{\sum_{j \in \mathbf{N}} x_j^r} V - x_i.$$

To ensure the existence of pure strategy equilibria in each such contest we impose the restriction $r \in \left(0, 1 + \frac{1}{N-1}\right]$. Applying standard procedure, the symmetric equilibrium is

$$x_i^*(N) = \frac{1}{N}\left(1 - \frac{1}{N}\right) rV.$$

The total expected effort is

$$E_s^*(N) = Nx_i^*(N) = \left(1 - \frac{1}{N}\right) rV. \tag{3}$$

The total expected output is

$$Y_s^*(N) = Nx_i^{*r}(N) = N\left[\frac{1}{N}\left(1 - \frac{1}{N}\right) rV\right]^r. \tag{4}$$

3. A TWO-STAGE CONTEST

Following Amegashie (1999), we consider the following two-stage contest. In the preliminary contest, $g \geq 1$ finalists are to be chosen from the N contestants, where $g \leq N$. There are N_j (>1) contestants in group j, $j = 1, \ldots, g$. We have $\Sigma_j N_j = N$. Every contestant competes against the other $N_j - 1$ contestants in his group j for the ticket to the final stage. After the preliminary competitions that select the g finalists, they enter the second-round competition. The winner receives the prize V. For the time being, we assume that the number of finalists chosen from the preliminary competition is given, which means g is exogenous.

We solve the game by backward induction. The second-stage competion among the g finalists can be solved as a standard single-period winner-take-all contest. As in Section 2, the second-stage equilibrium effort is

$$x_g^*(g) = \frac{1}{g}\left(1 - \frac{1}{g}\right) rV.$$

The total expected effort of the second period is

$$E_g^*(g) = gx_g^*(g) = \left(1 - \frac{1}{g}\right) rV. \tag{5}$$

The expected payoff of a finalist is

$$\pi_g^* = \frac{V}{g}\left[1 - \left(1 - \frac{1}{g}\right) r\right].$$

The total expected output of the second period is

$$Y_g^*(g) = gx_i^{*r}(g) = g\left[\frac{1}{g}\left(1 - \frac{1}{g}\right) rV\right]^r. \tag{6}$$

In the preliminary stage, the prize is simply π_g^* for every subgroup j. Similarly, we can solve for the equilibrium effort and output as below:

$$x_{j,1}^{*}(N_j)=\frac{1}{N_j}\left(1-\frac{1}{N_j}\right)r\pi_g^{*}.$$

The total expected effort of first period from group j is

$$E_{j,1}^{*}(N_j)=N_j x_{j,1}^{*}(N_j)=\left(1-\frac{1}{N_j}\right)r\pi_g^{*}.$$

The total expected output of the first period from group j is

$$Y_{j,1}^{*}(N_j)=N_j x_{j,1}^{*r}(N_j)=N_j\left[\frac{1}{N_j}\left(1-\frac{1}{N_j}\right)r\pi_g^{*}\right]^{r}.$$

The total expected effort of first period is

$$E_1^{*}=\sum_j E_{j,1}^{*}(N_j)=r\pi_g^{*}\sum_j\left(1-\frac{1}{N_j}\right). \tag{7}$$

The total expected output of the first period is

$$Y_1^{*}=\sum_j Y_{j,1}^{*}(N_j)=(r\pi_g^{*})^{r}\sum_j\left[\frac{1}{N_j}\left(1-\frac{1}{N_j}\right)\right]^{r}. \tag{8}$$

3.1. *Effort comparison*

We start the analysis of the optimal contest structure from the perspective of effort maximization. Combining equations 5 and 7, the grand total effort is

$$E_t^{*}=E_g^{*}(g)+E_1^{*}=rV\left\{\left(1-\frac{1}{g}\right)+\frac{1}{g}\left[1-\left(1-\frac{1}{g}\right)r\right]\sum_j\left(1-\frac{1}{N_j}\right)\right\}. \tag{9}$$

Lemma 2. *Given g, E_t^{*} is maximized when $N_j=k=\frac{N}{g}$, $\forall j$.*

Proof. We ignore the integer problem of N_j. Note $\pi_g^{*}>0$, and, thus, $\frac{1}{g}\left[1-\left(1-\frac{1}{g}\right)r\right]>0$. This is guaranteed by the range of r. Clearly, E_t^{*} is maximized when $\Phi=\sum_j\left(1-\frac{1}{N_j}\right)$ is maximized subject to $\Sigma_j N_j=N$, $N_j\geq 1$. We have $\frac{\partial\Phi}{\partial N_j}=\frac{1}{N_j^2}$, which strictly decreases with N_j. Whenever $N_{j1}>N_{j2}$, $\frac{\partial\Phi}{\partial N_{j1}}<\frac{\partial\Phi}{\partial N_{j2}}$. We can rebalance N_{j1} and N_{j2} to make N_{j1} lower and N_{j2} higher; this would lead to higher Φ.

According to Lemma 2, to determine whether an optimally-designed two-stage contest would increase the expected effort, we can focus on subgroups of even size. Note that when $g = 1$ or N, the two-stage contest coincides with the single-stage contest. Thus, an optimally-designed two-stage contest cannot be strictly worse than a single-stage contest.

When $N_j = k = \frac{N}{g}, \forall j$, the grand total effort is

$$E_t^* = E_g^*(g) + E_1^* = rV\left\{\left[\left(1-\frac{1}{g}\right)+\left(1-\frac{1}{k}\right)\right]-r\left(1-\frac{1}{k}\right)\left(1-\frac{1}{g}\right)\right\}. \tag{10}$$

When k is an integer, we say that the preliminary elimination stage with subgroup size k is feasible.

Comparing equations 3 and 10, we see that when $r = 1, E_t^* = E_s^*$. Note that $\left[\left(1-\frac{1}{g}\right)+\left(1-\frac{1}{k}\right)\right]-r\left(1-\frac{1}{k}\right)\left(1-\frac{1}{g}\right)$ strictly decreases with r when $g \neq 1$ and $g \neq N$; thus,

$$E_t^* > (<)\, E_s^* \Leftrightarrow r < (>) 1, \forall 2 \leq g \leq N-1. \tag{11}$$

Note that equation 11 holds for any g such that $2 \leq g \leq N - 1$. We see that when $r > 1$, adding a preliminary elimination stage never increases the expected effort. When $r < 1$, any non-trivial preliminary elimination contest with even subgroups would increase the total expected effort. These results are summarized in the following proposition.

Proposition 1. *(i) When $r > 1$, a preliminary elimination stage can never induce more expected effort; and (ii) when $r < 1$, any feasible non-trivial preliminary elimination contest with even subgroups would increase total expected effort.*

That is, to maximize total effort, the contest organizer will prefer a two-stage contest if and only if $r < 1$. She is indifferent between the two contest designs when $r = 1$.

The results of effort-maximizing contests have been carefully studied by Gradstein and Konrad (1999), who make an effort comparison between more general pairwise contests and simultaneous contests. In the following section, we will make a comparison in terms of total output.

3.2. *Output comparison*

Recall that when $g = 1$ or N, a two-stage contest is equivalent to a single-stage contest. This means that an optimally-designed contest can never be strictly worse than a single-stage contest.

In Subsection 3.1, we showed that a preliminary stage can only help when $r < 1$ if expected effort is the concern. In this subsection, we show that an optimally-designed preliminary stage can further help when $r > 1$ if, instead, output is the concern. For this purpose, we consider a preliminary stage with $g \geq 2$ even subgroups. Using the same procedure as for Lemma 2, we can also show that Y_t^* is maximized when $N_j = k = \frac{N}{g}$, $\forall j$. The total expected output of the first period is $Y_1^* = gY_{j,1}^*(k) = (r\pi_g^*)^r gk\left[\frac{1}{k}\left(1-\frac{1}{k}\right)\right]^r$. Therefore, the grand total output is

$$Y_t^* = Y_g^*(g) + Y_1^* = (rV)^r \left\{ g\left[\frac{1}{g}\left(1-\frac{1}{g}\right)\right]^r + N\left[\frac{(N-g)[g-r(g-1)]}{N^2 g}\right]^r \right\}. \tag{12}$$

To compare the total output of a two-stage contest with a single-stage contest, let

$$\begin{aligned}\Delta(r) &= (Y_s^* - Y_t^*)/(Vr)^r \\ &= N\left(\frac{N-1}{N^2}\right)^r - N\left[\frac{(N-g)[g-r(g-1)]}{N^2 g}\right]^r - g\left[\frac{g-1}{g^2}\right]^r.\end{aligned}$$

Lemma 3. *(i) For fixed N and g,* $\lim_{r\to 0^+}\Delta(r) = -g$; *(ii) for fixed* $g \geq 2$, *when* $N \to \infty$, $\Delta(r) \to +\infty, \forall r < 1$; and *(iii) for fixed* $g \geq 2$, *when* $N \to \infty$, $\Delta(r) \to -g\left[\frac{g-1}{g^2}\right]^r, \forall r > 1$.

Proof. Clearly, for fixed N and g, $\lim_{r\to 0^+}\Delta(r) = -g\left[\frac{g-1}{g^2}\right]^r = -g$. For fixed $g \geq 2$, when $N \to \infty$, the order of $\Delta(r)$ is $N^{1-r}\{1-[1-r(1-1/g)]^r\} - g\left[\frac{g-1}{g^2}\right]^r$. Thus $\Delta(r) \to -g\left[\frac{g-1}{g^2}\right]^r < 0$ when $r > 1$ and $\Delta(r) \to +\infty$ when $r < 1$.

Lemma 3 leads to the results in the following proposition.

Proposition 2. *(i) For fixed N and g, there is a small right neighbourhood of* $r = 0$ *such that adding a preliminary stage with g even subgroups leads to more expected output. (ii) Let* $N = kg$, *where* $g \geq 2$. *For any* $r \in (0,1)$, *adding a preliminary stage with g even subgroups leads to less expected output when k is sufficiently large.*

When $r \in (0, 1)$, Proposition 1 shows that any feasible non-trivial preliminary elimination contest with even subgroups would increase the total expected effort. Proposition 2 demonstrates that adding a preliminary stage may have an opposite impact on expected output if the productivity level is sufficiently small. However, when the number of contestants is large, adding a preliminary stage with even grouping may, indeed, negatively affect the expected output.

Corollary 1. *For any $N \geq 4$, there is a small right neighbourhood of $r = 0$ such that adding an optimaly-designed preliminary stage leads to more expected output.*

Proof. If N is an even number, take $g = N/2$. Proposition 2 implies that the claim is true.

If N is an odd number, take $g = (N - 1)/2$. Assign 2 contestants to the first $g - 1$ groups and 3 contestants to the last group. It is easy to verify that the group with 3 contestants generates more output than any group with 2 contestants. Thus, the contest generates more output than a contest with $N - 1$ contestants who are evenly divided into $g = (N - 1)/2$ groups. Proposition 2 can be applied to the later contest for the existence of the small neighbourhood of $r = 0$ such that the original grouping leads to higher output. Clearly, the obtained neighbourhoods apply to the optimal grouping (the optimal g and the optimal sizes of the subgroups) in the preliminary stage.

Lemma(*iii*) suggests that for any fixed $r > 1$ when N is large, adding a preliminary stage with even grouping definitely increases the expected output. However, as N increases, the eligible upper bound for r, (i.e. $1+\frac{1}{N-1}$) decreases.[6] For any fixed $r > 1$, we may end up with $r > 1+\frac{1}{N-1}$ for an N that is sufficiently large, as required by Lemma 3(*iii*). Nevertheless, it is still true that for any $g \geq 2$, when N is sufficiently large, we can find a small right neighbourhood of $r = 1$ that depends on N, such that as long as r falls into this neighbourhood, adding a preliminary stage with even groups, indeed, increases the expected output.

The arguments are more elaborate and rely on the property of $\frac{d\Delta(r)}{dr}$ at $r = 1$. Note:

$$\frac{d\Delta(r)}{dr} = N\left(\ln\frac{N-1}{N^2}\right)\left(\frac{N-1}{N^2}\right)^r - g\left(\ln\frac{g-1}{g^2}\right)\left(\frac{g-1}{g^2}\right)^r - N\left[\frac{(N-g)[g-r(g-1)]}{N^2 g}\right]^r$$
$$\left\{\ln(N-g) - \ln N^2 g + \ln[g-r(g-1)] + \frac{r(1-g)}{g-r(g-1)}\right\}.$$

Lemma 4. *For fixed $g \geq 2$, when $N \to \infty$,* $\left.\frac{d\Delta(r)}{dr}\right|_{r=1} \to -\infty$.

[6] Recall that our analysis is restricted to pure strategies; that is, $r \leq 1+\frac{1}{N-1}$. We require a nonnegative expected payoff in both contest structures.

Proof. For fixed $g \geq 2$, when $N \to \infty$, the order of $\left.\frac{d\Delta(r)}{dr}\right|_{r=1}$ is $\left(\frac{1}{g}-1\right)\ln N \to -\infty$.

Note that when $r = 1$, we have $\Delta(1) = 0$. Thus, Lemma 4 means that for any given g, when the number of contestants (N) is sufficiently large, there exists a small ε_N such that when $r \in (1, 1 + \varepsilon_N)$, $\Delta(r) < 0$. In other words, there exist convex production functions such that adding a preliminary stage with even subgroups of size N/g would increase the total expected output when N is sufficiently large.

PROPOSITION 3. *When N is sufficiently large, there exists a small ε_N such that when $r \in (1, 1 + \varepsilon_N)$, the optimally-designed preliminary stage leads to higher expected output.*

Proof. Take $g = 2$. When N is an even number, Lemma 4 means that there exists a small ε_N such that when $r \in (1, 1 + \varepsilon_N)$, adding a preliminary stage with even subgroups of size $N/2$ would increase the total expected output.

When N is an odd number, apply Lemma 4 to $N - 1$ and $N + 1$, which are even numbers. There exists a common neighbourhood $(1, 1 + \varepsilon)$ such that when $r \in (1, 1 + \varepsilon)$, adding a preliminary stage with two even subgroups would increase the total expected output. Consider the following grouping of N contestants: Assign $\frac{N-1}{2}$ contestants to group 1, and $\frac{N+1}{2}$ contestants to group 2. Clearly, the expected output from this grouping dominates the smaller one of the outputs when there are $N - 1$ and $N + 1$ contestants who are evenly divided into two subgroups. Thus, when $r \in (1, 1 + \varepsilon)$, the expected output from this grouping dominates that from a single-stage contest.

Clearly, the obtained neighbourhoods apply to the optimal grouping (the optimal g and the optimal sizes of the subgroups) in the preliminary stage.

Proposition 3 illustrates a new insight into the desirability of a preliminary stage in output maximization, which diverges from the insight revealed by Proposition 1 for effort maximization. While Proposition 1 shows that adding a preliminary stage can never benefit expected effort when the production function is convex, Proposition 3 shows that even with a convex production function, adding an optimally-designed preliminary stage can still benefit expected output when the number of contestants is big.

We now briefly interpret this divergence. The exponent r measures the productivity of the production process. When r is smaller than one, a two-stage contest will induce higher total effort and a single-stage contest will induce higher total output, because a concave production technology transfers effort into output compressively. If r is greater than one, however, a two-stage contest will result in less total effort but higher total output than a single-stage contest, because a convex production technology expansively transfers effort into

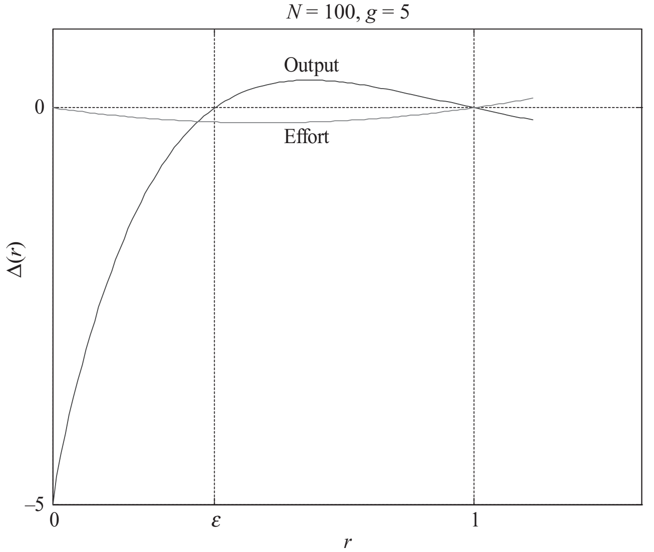

Figure 1. Effort versus output maximization

output. Moreover, when productivity is so small that it converges to zero, with each contestant's equilibrium effort $x \to 0$, output $x^r \to 1$, adding a preliminary stage always induces higher value in terms of both effort and output, because no matter how much effort players expend, they will converge to a constant output level of one. Therefore, for a fixed number of contestants, the more stages, the higher total output. The desirability of an elimination stage depends crucially on the productivity of the effort; adding a preliminary stage can improve output for both concave and convex production functions.

Our results provide contest organizers with guidelines for the optimal contest design. With a large pool of high-producing contestants, an output-maximizing contest organizer should add a preliminary stage and divide contestants into even groups; however, when contestants are lower-producing, a pooling contest will be the optimal choice. These results are in contrast to the insight on effort maximization, which holds that adding a preliminary stage can never benefit expected effort when the production function is convex. With a convex impact function, a more discriminatory contest means that a better performance can be translated into a higher likehood of winning. Faced with a larger number of competitors, each contestant will bid more aggressively in a one-stage pooling contest, which elicits higher total expected effort.

To compare the differences between these maximization targets, Figure 1 illustrates one such case.

4. CONCLUSION

This paper studies the issue of optimal structure in contests. In contrast to previous literature, to attain maximum effort by contestants, we focus on a structural design that maximizes total expected output. In a winner-take-all contest framework, we explore whether the contest organizer would benefit by adding a preliminary elimination stage for output maximization. Our results indicate that a two-stage contest can improve output for both concave and convex production functions, which is the opposite of the insight on effort maximization. These results can also easily be extended to multi-stage contests for output maximization.

In this paper, we assumed that the number of finalists in the preliminary stage is given. In many situations, it is more realistic to assume that the organizer rather endogenously chooses the number of finalists, g, in the preliminary stage. Providing the continuity of g, although analytical diffculties prevent us from solving the form explicitly, we can show that there must exist an optimal number of shortlists that maximize total output in these two stages, as discussed in Amegashie (1999).

Furthermore, our setting (i.e. adding a preliminary stage) is only one form of output-maximization contest design. Other examples include the settings of Appelbaum and Katz (1987), who study the optimal prize size, Baye *et al.* (1993), who focus on the optimal admittance to a contest, and Moldovanu and Sela (2001), who investigate the optimal allocation of multiple nonidentical prizes. Various issues related to the optimal design of contests in these diverse settings remain open, and they deserve to be explored seriously in the future.

REFERENCES

Amegashie, J. A. (1999) 'The Design of Rent-Seeking Competitive Committees, Preliminary and Final Contests', *Public Choice* 99, 63–76.

Amegashie, J. A. (2000) 'Some Results on Rent-Seeking Contests with Shortlisting', *Public Choice* 105, 245–53.

Amegashie, J. A. (2003) 'Burning out in Sequential Elimination Contests', Working Paper No. 2002-8, University of Guelph.

Appelbaum, E. and E. Katz (1987) 'Seeking Rents by Setting Rents', *Economic Journal* 97, 685–99.

Baik, K. H. and I.-G. Kim (1997) 'Delegation in Contests', *European Journal of Political Economy* 13, 121–30.

Baik, K. H. and S. Lee (2000) 'Two-Stage Rent-Seeking Contests with Carryovers', *Public Choice* 103, 285–96.

Baik, K. H. and J. Shogren (1995) 'Contests with Spying', *European Journal of Political Economy* 11, 441–51.

Baye, M. R., D. Kovenock and C. G. de Vries (1993) 'Rigging the Lobbying Process: An Application of the All-Pay Auction', *American Economic Review* 83, 289–94.

Clark, D. J. and C. Riis (1996) 'A Multi-Winner Nested Rent-Seeking Contest', *Public Choice* 87, 177–84.

Fu, Q. and J. Lu (2012) 'The Optimal Multi-Stage Contest', *Economic Theory* 5, 351–82. Reviewed by NAJ Economics.

Gradstein, M. (1998) 'Optimal Contest Design: Volume and Timing of Rent Seeking in Contests', *European Journal of Political Economy* 14, 575–85.

Gradstein, M. and K. A. Konrad (1999) 'Orchestrating Rent Seeking Contests', *Economic Journal* 109, 535–45.

Higgins, R. S., W. F. Shughart and R. D. Tollison (1985) 'Free Entry and Efficient Rent-Seeking', *Public Choice* 46, 247–58.

Katz, E. and J. Tokatlidu (1996) 'Group Competition for Rents', *European Journal of Political Economy* 12, 599–607.

McFadden, D. (1973) 'Conditional Logit Analysis of Qualitative Choice Behavior', in P. Zarembka (ed.), *Frontier in Econometrics*, New York: Academic.

McFadden, D. (1974) 'The Measurement of Urban Travel Demand', *Journal of Public Economics* 3, 303–28.
Moldovanu, B. and A. Sela (2001) 'The Optimal Allocation of Prizes in Contests', *American Economic Review* 91, 542–58.
Rosen, S. (1986) 'Prizes and Incentives in Elimination Tournaments', *American Economic Review* 76, 701–15.
Stein, W. and A. Rapoport (2005) 'Symmetric Two-Stage Contests with Budget Constraints', *Public Choice* 124, 309–28.
Tullock, G. (1980) 'Efficient Rent Seeking', in J. M. Buchanan, R. D. Tollison and G. Tullock (eds), *Toward a Theory of the Rent-Seeking Society*. College Station: Texas A&M University Press.